QUESTIONS (I).

Please solve question (1), and then choose and solve as many of the questions (2) to (8) listed below as you can.

1 a) Find the derivative of the following functions:

$$i)\ \ f(x) = 4x^{-1/4} + \frac{x}{(x^2-1)^{1/3}};\quad ii)\ \ f(x) = \exp[\sin^2(1/x)]$$

b) Solve the following integrals:

$$i)\ \int x \sin x\ dx\ ;\qquad ii)\ \int \frac{1}{\sqrt{1-x^2}}\ dx$$

Numbers.

2 Find all the numbers x for which:
(i) $4 - x < 3 - 2x$
(ii) $|x - 3| = 8$

3 Prove that if: $|x - x_0| < \epsilon/2$ and $|y - y_0| < \epsilon/2$ then

$$|(x + y) - (x_0 + y_0)| < \epsilon \quad \text{and} \quad |(x - y) - (x_0 - y_0)| < \epsilon$$

Geometry and Dynamics.

4 Start with a square where each side is one unit long and cut out four identical square corners, folding in the four edges to make an open-topped box (see Figure 1). What size should the corners x be to maximize the volume of the box?

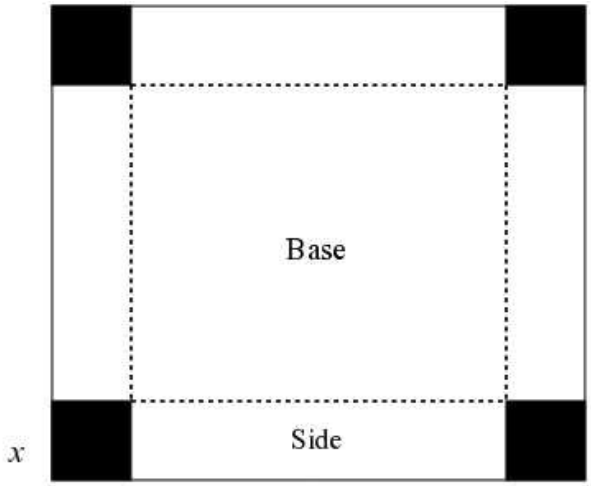

Figure 1

5 A particle of mass m has potential energy

$$V(x) = \frac{K}{mx^2} - \frac{G}{x}$$

where x is a distance, and K and G are constants. Sketch the potential energy $V(x)$ as a function of x. Find the extrema of $V(x)$, if there are any, and in that case indicate whether they are maxima or minima. What types of motion are possible for this particle?

6 Suppose that A, B, and C are three points in a plane, such that $AB = AC = BC = 1$. At each point in time, A is moving toward B, B is moving toward C, and C is moving towards A, all with speed $v = 50$.

Figure 2 shows how the movement will look like. The red curve represents the path of point A, the green curve - the path of point B, the blue curve the path of point C. At what time T will all the points reach the center of the triangle?

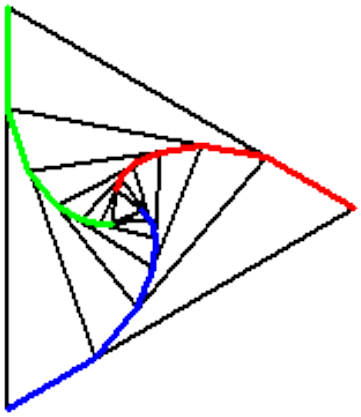

Figure 2

At what time T will all the points reach the center of the triangle?

Complex Numbers and Series.

7 Using the definition:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{if } k \neq 0, n$$

prove that

$$\sum_{j=0}^{n} \binom{n}{j} = 2^n$$

8 a) Find the absolute value and argument of the number $(3 + 4i)^{-1}$; then make a diagram of the complex plane indicate its location.